Abstract

Graphs defined on groups have a long history. Many graphs are defined with vertex set being a group G and the edge set reflecting the structure of G in some way, for example, Cayley graph, commuting graph, power graph, enhanced power graph etc. Recently in 2023, Cameron and Kuzma introduced a notion of deep commuting graph of a finite group, where the vertex set of the graph is the group G and two distinct vertices are adjacent if their preimages commute in a Schur cover of G. Add to this study, Arunkumar et al. introduced the notion of super graph on a group, by considering an equivalence relation on the elements of the group G and an existing graph structure on G.

In this thesis, we have considered some combinatorial properties of the above graphs defined on groups. We explored the notion of super graphs defined on groups via generalized join of $\mathcal{R}$-compressed graphs for an equivalence relation $\mathcal{R}$ defined on the vertex set of a graph. We also studied equality between order, conjugacy and equality super graphs. Then we explored the adjacency and Laplacian spectra of order super commuting graphs and conjugacy super commuting graphs of D_{2n}, Q_{4n} and the non-abelian group of order pq, denoted by $C_p \rtimes C_q$ for distinct odd primes p and q. In addition, we discussed the dominant vertices of the order super commuting graphs of the symmetric group S_n and the alternating group A_n and the connectedness and diameter of their respective reduced graphs. Next, we characterized certain cut sets of $\mathcal{P}(C_n)$ such that a minimum cut set must be one of them. We also investigated the vertex connectivity of $\mathcal{P}(C_n)$ for several values of n. Lastly, we concentrated on the deep commuting graph of finite groups, especially for D_{2n}, Q_{4n}, S_n, A_n and finite abelian groups. We explored the dominant vertices of Δ_D for the above groups and also investigated the cases for which the deep commuting graph coincides with the commuting graph or the enhanced power graph.