ABSTRACT

In modern Fourier analysis, the study of multilinear (m-linear) operators focuses on establishing their boundedness from the m-fold product of normed spaces to the appropriate normed spaces. For instance, Lebesgue spaces are commonly considered as the normed spaces in this context. These operators find applications in various fields, including partial differential equations, complex analysis, and quantum mechanics. A more comprehensive investigation consists of exploring weighted inequalities, which involve determining the boundedness of these operators on weighted Lebesgue spaces. Weighted inequalities hold significance in a broader scope, impacting areas such as vector-valued operators, operator extrapolation, and the theory of Laplace's equation boundary value problems on Lipschitz domains.

Over the past three decades, a parallel theory to classical Fourier analysis, associated with root systems and reflection groups, has emerged in Euclidean harmonic analysis. This theory, known as Fourier analysis in the Dunkl setting, serves as a generalization of classical Fourier analysis. Within this context, significant progress has been made, particularly in understanding singular integrals, Fourier multiplier operators, and potential-type operators. However, exploration of multilinear operators or weighted inequalities within the Dunkl setting has been relatively limited. The primary aim of this thesis is to delve into the weighted boundedness of some multilinear operators in the Dunkl framework.

The first result of this thesis is one and two-weight estimates for multilinear Calderón-Zygmund type singular integral operators in the Dunkl setting, along with the associated maximal operators. Importantly, these operators distinguish themselves from classical Calderón-Zygmund singular integral operators by incorporating both the 'Dunkl metric' and the usual metric in their definition. In the subsequent chapter, we initially establish Littlewood-Paley theory in the Dunkl framework, utilizing it to prove a Coifman-Meyer type bilinear multiplier theorem associated with the Dunkl transform. Additionally, we show that these multiplier operators are examples of multilinear Dunkl-Calderón-Zygmund operators and derive weighted estimates for them. In the final chapter, we study similar weighted inequalities for a different type of operators known as multilinear Dunkl fractional integral operators and multilinear fractional maximal operators.