ABSTRACT

Over the past few decades, Lipschitz spaces and their structural properties have received significant attention, especially in nonlinear functional analysis. An important result in this direction was obtained in 1964 by Joram Lindenstrauss [42, Theorem 2]. He proved the existence of a continuous linear projection from the Banach space $Lip_0(X, \mathbb{R})$ onto the linear dual space $L(X, \mathbb{R})$.

One of the primary aims of this thesis is to generalize the Lindenstrauss's result (particularly when the codomain is general Banach space Y) both in Lipschitz and two-Lipschitz cases. We study the structure of Lipschitz operator spaces and two-Lipschitz operator spaces, together with their quotient spaces. We also give a necessary and sufficient condition for a Lipschitz map to be linear, and for a two-Lipschitz map to be bilinear.

As an application, we identify ker($\beta_{\mathbb{R}}$) as the predual of the quotient space $L^\infty(\mathbb{R})/\tilde{\mathbb{R}}$ (respectively, $L^\infty(\mathbb{R}^2)/\tilde{\mathbb{R}}$), where $\tilde{\mathbb{R}}$ denotes the space of constant functions.

Motivated by classical results of Lindenstrauss we further investigate quotient spaces of the form $Lip_0(X)/\mathcal{A}$, where $\mathcal{A}$ is a finite-dimensional subspace. We prove that these quotients are dual spaces and provide an explicit description of their preduals.

We then focus on the space $Lip_0^{ph}(X)$ of positively homogeneous real-valued Lipschitz functions. This space satisfies

$$X^* \subsetneq Lip_0^{ph}(X) \subsetneq Lip_0(X).$$

We show that $Lip_0^{ph}(X)$ is a dual space and also the preannihilator of a closed subspace of the Lipschitz-free space. As a consequence, the quotient $Lip_0(X)/Lip_0^{ph}(X)$ is also a dual space. Moreover, with a suitable multiplication, $Lip_0^{ph}(X)$ becomes a Banach algebra.

Finally, we introduce the notion of extensively bounded mappings, which generalize Lipschitz mappings, and study the associated nonlinear operator ideals. With an appropriate modification in the linear setting, we define extensively p-summing mappings. We establish Pietsch-type domination and factorization theorems for these mappings. We also study extensively bounded finite-rank, compact, p-compact mappings, p-integral, p-nuclear operators and describe the corresponding operator ideals.