ABSTRACT

The Calderón problem has played a central role in the study of inverse problems and partial differential equations (PDEs) since its introduction by Alberto P. Calderón in 1980. Sylvester and Uhlmann made a major breakthrough in this direction for dimensions greater than or equal to three. They introduced the concept of complex geometrical optics (CGO) solutions in their work, which motivated more researchers to explore this area in higher dimensions. Since this inverse problem is mostly ill-posed, a better understanding of the stability becomes very important. However, the optimal stability estimates are of logarithmic type in these settings. This inspired the notion of increasing stability, where it was explored whether an improvement of the stability to Hölder type is possible in the presence of frequency.

In this thesis, we focus on high-frequency stability estimates for inverse boundary value problems associated with the Schrödinger equation, the biharmonic operator, and the polyharmonic operator with constant attenuation, working in the partial data setting where part of the boundary is flat. We first explore stability estimates for the linearized inverse problems related to the Schrödinger and biharmonic operators with constant attenuation for potentials in C^1. We then consider the more general polyharmonic case and derive stability estimates for less regular potentials in H^s with $0 < s < \frac{1}{2}$. This result also generalizes the results obtained in the earlier cases, where C^1 regularity was assumed for the potentials.

Next, we focus on stability estimates for the corresponding nonlinear inverse problems under the same low-regularity assumptions on the potential. While in the linearized problems our stability estimates exhibit a polynomial dependence on the frequency in all cases, for the nonlinear problems we are able to establish such polynomial dependence only for the Schrödinger equation. For the biharmonic and polyharmonic operators, the dependence is exponential, which is consistent with the work of Liu (2020) in the absence of attenuation.

Ajith Kumar T