Abstract

This thesis investigates graph deletion problems under various connectivity constraints, focusing on four key variants.

1. Connected Cluster Vertex Deletion: We study the *Connected Cluster Vertex Deletion* (CCVD) problem: given a graph G and integer k, delete at most k vertices to obtain a cluster graph, with the deleted set inducing a connected subgraph. We first give a baseline $\mathcal{O}(6^k \cdot n^{\mathcal{O}(1)})$-time algorithm and improve it to $\mathcal{O}(5.236^k \cdot n^{\mathcal{O}(1)})$ via refined branching. We also show that CCVD admits no polynomial kernel unless NP $\subseteq$ coNP/poly.

2. Component Size Restrictions: We examine Feedback Vertex Set (FVS) variants that limit component sizes, proving FPT results for At-most-c and At-least-c FVS while establishing kernelization lower bounds.

3. Highly Connected Solutions: We ensure that after deletion, the solution remains well-connected. We establish fixed-parameter tractability (FPT) for problems such as Vertex Cover (VC), Feedback Vertex Set (FVS), Dominating Set (DS), and Multiway Cut, while demonstrating the intractability of the Vertex Multicut problem.

4. Transitive-Free Graphs: We explore deletion problems in tournaments and other directed graph classes, where every arc is crucial for connectivity, analyzing polynomial kernels and complexity hardness.

Our findings provide new structural insights and algorithmic techniques, contributing to the study of connectivity-constrained problems and paving the way for efficient parameterized algorithms.