ABSTRACT

In this thesis, we explored graph partitioning and connectivity problems, mainly within the scope of parameterized complexity theory. In a graph partitioning problem the goal is to divide the edge set/vertex set or both subject to a set of constraints. The first problem we studied is Budgeted Graph Coloring, which falls under graph partitioning. It involves partitioning the vertices of a graph into independent sets, respecting specific size constraints outlined by a budget vector $(b_1, b_2, \cdots, b_c)$. Our research spans over hardness results, polynomial-time algorithms for various graph classes, and fixed-parameter tractability (FPT) results for structural parameters such as vertex cover and distance to clique. The second problem, Constrained k-way Cut($\mathscr{F}$), approaches graph partitioning differently. By deleting at most s edges, we aim to partition the vertex set into exactly k components belonging to a given family $\mathscr{F}$ of graphs such that each component is an induced subgraph of G. We extensively studied these problems across different graph families like trees, bipartite, chordal, and bounded degree graphs. Additionally, we looked into a problem merging connectivity aspects with graph partitioning: Shortest Non-Separating Path. It involves finding a path of length k between a pair of vertices whose removal maintains graph connectivity. We proved this problem to be intractable parameterized by k. Conversely, Shortest Non-Disconnecting Path, aiming for a connected graph after removing the edges of such a path, turned to be FPT for the same parameter, using matroid-based tools. Conversely, graph connectivity problems involve preserving connections between pairs or sets of vertices under certain conditions. We began with (A, ℓ)-Path Packing, seeking k disjoint paths of exactly ℓ, with endpoints restricted to a given subset A. We demonstrate the intractability using a randomized reduction for a large parameter. Since this reduction can be very adaptable for other similar problems, it can be of independent interest. Finally, we explored Shortest Path with Forcing Graph, a problem involving a graph and an auxiliary graph called the forcing graph which depicts a set of constraints. The objective is to find a set of edges in the original graph which is a vertex cover in the forcing graph as well as contains an (s, t)-path in the original graph. We designed a polynomial kernel for the parameter k and improved it by imposing constraints on the original or forcing graph.