Summary

In this thesis, we look into the near-equilibrium dynamics of two very familiar systems with well-known thermodynamic properties: black holes and fluids. Part I explores the effect of reparametrizations of the horizon's null generators on the entropy production on the horizon of a black hole in Einstein-Gauss-Bonnet theory. Part II attempts to understand the relationship between stability and causality in two well-known stable-causal models of relativistic hydrodynamics: the Müller-Israel-Stewart (MIS) model and the Bemfica-Disconzi-Noronha-Kovtun (BDNK) model, first by Lorentz transforming to ultra-high boosted frames, and then by field redefinitions of the thermodynamic variables. In all the cases, analysis was performed up to the linearized order in amplitude dynamics.

Recent advances in the second law of black hole thermodynamics for higher-derivative gravity theories have shown that there exists an entropy density and an entropy current on the dynamical horizons of black holes of these theories, which, by construction, have a total non-negative divergence for linearized amplitude perturbations about a stationary solution. However, the formulation of this entropy density and current depends on the spatial slicing of the horizon along its affinely-parametrized null generators.

In the first work 3 of Part 1, we study the non-trivial changes in entropy density and current under a local reparametrization of the affinely-parametrized null-generators to another family of affinely-parametrized null-generators. We find that the entropy density and entropy current change such that their divergence, and hence the net entropy production on the horizon, remain invariant.

In the second work 4, we dualize this entropy density and entropy current to an entropy current for a fluid residing on the boundary of an asymptotically AdS Einstein-Gauss-Bonnet black-brane solution. The boundary coordinates used to describe the fluid's entropy current correspond to a non-affine parametrization of the null generator on the horizon. Although the Gauss-Bonnet

coupling doesn't lead to any corrections to the fluid entropy current in the first order in boundary-derivative expansion, there are non-trivial corrections in the second order dependent on the horizon-to-boundary mapping functions, which aren't necessarily expressible solely in terms of fluid variables. Hence, we conclude that for generic situations, the boundary entropy current thus obtained doesn't admit a derivative expansion.

One of the most difficult challenges in relativistic hydrodynamics has been to formulate hydrodynamic theories that admit perturbations about local equilibrium that are causal (i.e., do not exit the light cone) and stable (i.e., decay down with time). The decades-old MIS and the recently developed BDNK are two such formalisms with some regions in their parameter space where the theories are stable and causal.

The first work 5 of Part 2 investigates the connection between stability and causality properties using these two theories as case studies. Here, we utilize linearized stability analysis to obtain the causality criteria for these two theories unambiguously. We find that the regions of the parameter spaces of both these theories which are stable at an ultra-high boost (i.e., boost velocity = speed of light), are stable at all other boost velocities and, hence, causal. The causality criteria thus obtained from a low-wavenumber analysis match the asymptotic causality criteria performed at a high-wavenumber of the theories.

In the second work of this part 6, we rewrite the conformal BDNK stress tensor in the "Landau frame" by redefining the temperature and velocity fields. We show that to maintain stability and causality in the "Landau frame", one either has to have an infinite number of derivative corrections or has to include new 'non-fluid' variables in the formalism. Moreover, we find that this incorporation of 'non-fluid' variables is a non-unique procedure.